Superluminal effects and negative group delays in electronics, and their applications

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(Received 15 June 2002; published 12 November 2002)

The causality principle does not forbid negative group delays of analytic signals in electronic circuits; in particular, the peak of a pulse can leave the exit port of a circuit before it enters the input port. Furthermore, pulse distortion for these "superluminal" analytic signals can be negligible in both the optical and electronic domains. Here we suggest a possible extension of these ideas to microelectronics. The underlying principle is that negative feedback can be used to produce negative group delays. Such negative group delays can be used to cancel out the positive group delays introduced by transistor latency, as well as the propagation delays due to the interconnections between transistors. Using this principle, it may be possible to speed up computer systems.

DOI: 10.1103/PhysRevE.66.056601 PACS number(s): 41.20.−q, 03.65.Sq, 42.25.Bs

I. INTRODUCTION

The existence of faster than c-group velocities was predicted by Garrett and McCumber [1]. The first experimental verification was performed by Chu and Wong [2], and later reproduced in the millimeter range of the electromagnetic spectrum by Segard and Macke [3]. The former experiment showed that picosecond laser pulses propagated superluminally through an absorbing medium in the region of anomalous dispersion inside the optical absorption line. There are also experimental results, showing that the process of photon tunneling in quantum physics is superluminal [4].

Recent optical experiments by Wang *et al.* [5] have verified the prediction [6] that superluminal pulse propagation can occur in transparent media with optical gain. These experiments showed that a laser pulse can propagate with little distortion in an optically pumped cesium vapor cell with a group velocity greatly exceeding the vacuum speed of light. In fact, the group velocity for the laser pulse in this experiment was observed to be negative. The peak of the output laser pulse left the output face of the cell before the peak of the input laser pulse entered the input face of the cell.

These counterintuitive pulse sequences were also seen to occur in experiments on electronic circuits [7]. The first of these experiments utilized a circuit consisting of an operational amplifier with a passive RLC network in a negative feedback loop. This circuit produced a negative group delay in which the peak of the output voltage pulse left the output port of the circuit before the peak of the input voltage pulse entered the input port of the circuit. Such a seemingly noncausal phenomenon does not, in fact, violate the principle of causality, since there is sufficient information in the early portion of any analytic voltage wave form to reproduce the entire waveform earlier in time. Furthermore, it was shown that causality is solely connected with the occurrence of discontinuities in a signal (e.g., "fronts" and "backs"), and not with the peaks in the voltage waveform [8].

We believe that these counterintuitive ideas can be applied to the design of microelectronic devices. At least two problems that may be relevant are transistor latency (i.e., the finite RC rise time of metal-oxide-semiconductor field-effect transistors caused by their intrinsic gate capacitance), as well as propagation delays due to the RC time constants in the interconnections between individual transistors. This latter problem may become significant as microprocessor circuits are increasingly reduced in size; in particular, as the transistor switching time becomes increasingly faster, the propagation delay from transistor to neighboring transistor becomes relatively longer [9].

The propagation delays of interconnections arise from a combination of the resistivity of the evaporated metal wire connecting two nearby transistors, and the dielectric constant of the insulator which supports the interconnecting wire. One of the current solutions to this problem is to use copper interconnections instead of aluminum (which has traditionally been used). Another solution is to use insulators with a lower dielectric constant to support the interconnecting wires. These steps reduce propagation delays, but do not eliminate them altogether. Here we suggest a radically different approach which, in principle, can eliminate these kinds of delays by introducing compensatory negative group delays. For example, it may be possible to eliminate the positive propagation delay from an interconnect by exactly compensating with an equal, but opposite, negative group delay.

On computer chips, there is a well-known effect, known as "clock skew," related to the time synchronization of logic pulses at some final logic gate. If different pulses are routed through different paths, they will, in general, arrive at different times. This deleterious effect prevents the use of higher clock rates, because extra delays must typically be deliberately added to early arriving pulses, to force all pulses to arrive simultaneously at the final gate [9]. Our compensation

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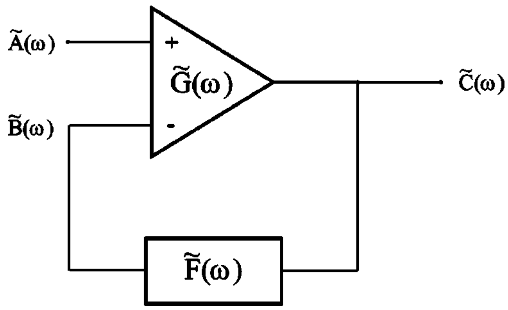

FIG. 1. Operational amplifier circuit with negative feedback.

scheme should lead to a path independence for the routing time of logic pulses throughout the computer system, producing a novel solution to the clock synchronization problem. Since there would no longer be any appreciable delays for a pulse to propagate from one logic gate to the next, the routing time of a logic pulse to a final logic gate could become largely independent of the path taken by this pulse inside the computer.

II. GENERAL PRINCIPLES FOR GENERATING NEGATIVE GROUP DELAYS

A. Negative group delays necessitated by the golden rule for operational amplifier circuits with negative feedback

In Fig. 1, we show an operational amplifier with a signal entering the noninverting $(+)$ port of the amplifier. The output port of the amplifier is connected back to the inverting $(-)$ port of the amplifier by means of a black box, which represents a passive linear circuit with an arbitrary complex transfer function $\tilde{F}(\omega)$. We thus have a linear amplifier circuit with a negative feedback loop containing a passive filter. In general, the transfer function of any passive linear circuit, such as a RC low-pass filter, will always lead to a positive propagation delay through a circuit.

However, for operational amplifiers with a sufficiently high gain-feedback product, the voltage difference between the two input signals arriving at the inverting and noninverting inputs of the amplifier must remain small at all times. The operational amplifier must, therefore, supply a signal with a negative group delay at its output, such that the positive delay from the passive filter is exactly canceled out by this negative delay at the inverting $(-)$ input port. The signal at the inverting $(-)$ input port will then be nearly identical to that at the noninverting $(+)$ port, thus satisfying the golden rule for the voltage difference at all times. The net result is that this negative feedback circuit can produce an output pulse whose peak leaves the output port of the circuit before the peak of the input pulse arrives at the input port of this circuit.

In Fig. 2, we show experimental evidence for this counterintuitive behavior in the special case of an RLC tuned bandpass circuit in the negative feedback loop [7]. The peak of an output pulse is advanced by 12.1 ms relative to the input pulse. The output pulse has obviously not been significantly distorted with respect to the input pulse by this linear circuit. Also, note that the size of the advance of the output

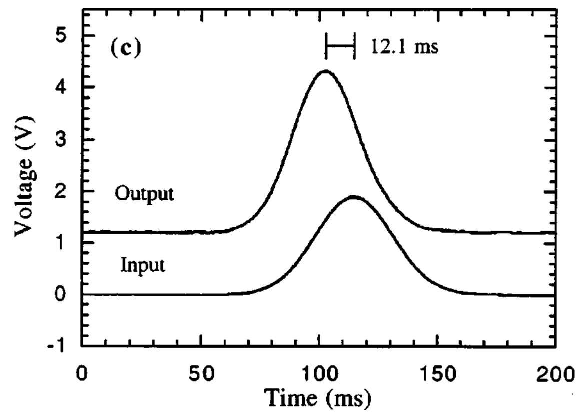

FIG. 2. Experimental results showing the pulse advancement.

pulse is comparable in magnitude to the width of the input pulse.

A second experiment demonstrated that causality is not violated in this process; when the input signal voltage was suddenly shorted to zero, the output was also reduced to zero at essentially the same instant. The result is shown in Fig. 3. This demonstrates that the circuit cannot advance in time truly discontinuous changes in voltages, the only points on the signal wave form which are connected by causality [8]. However, for the analytic changes of the input signal wave form, such as those in the early part of the Gaussian input pulse, the circuit evidently has the ability to extrapolate the input wave form into the future in such a way as to reproduce the peak of the output Gaussian pulse before the input peak has arrived. In this sense, the circuit anticipates the arrival of the Gaussian pulse.

B. The golden rule and the inversion of the transfer function of any passive linear circuit

Now we shall analyze under what conditions the golden rule holds and negative group delays are produced. In Fig. 1, $\tilde{A}(\omega)$ denotes the complex amplitude of an input signal of frequency ω into the noninverting $(+)$ port, and $\tilde{B}(\omega)$ refers to that of the feedback signal into the inverting $(-)$ port of

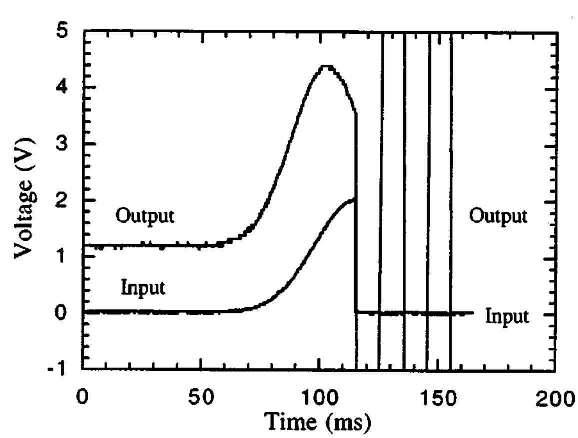

FIG. 3. Experimental results showing that discontinuities cannot be advanced.

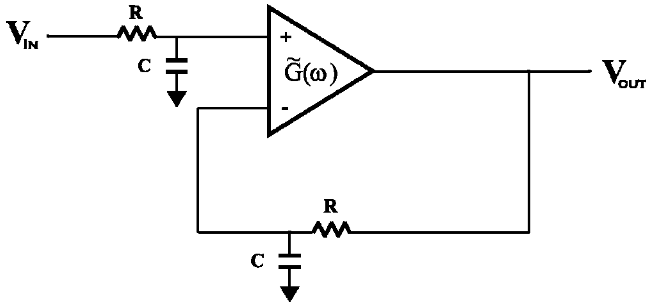

FIG. 4. Circuit with the RC filter placed before the negative feedback circuit.

the amplifier. The output signal $\tilde{C}(\omega)$ is then related to the feedback signal $\tilde{B}(\omega)$ by means of the complex linear feedback transfer function $\tilde{F}(\omega)$. The voltage gain of the operational amplifier is characterized by the active complex linear transfer function $\tilde{G}(\omega)$, which amplifies the difference of the voltage signals at the $(+)$ and $(-)$ inputs to produce an output signal as follows:

$$\tilde{C}(\omega)=\tilde{G}(\omega)[\tilde{A}(\omega)-\tilde{B}(\omega)]. \tag{1}$$

Defining the total complex transfer function $\tilde{T}(\omega) \equiv \tilde{C}(\omega)/\tilde{A}(\omega)$ as the ratio of the output signal $\tilde{C}(\omega)$ to input signal $\tilde{A}(\omega)$, we obtain for the total transfer function

$$\tilde{T}(\omega)=\frac{\tilde{G}(\omega)}{1+\tilde{F}(\omega)\tilde{G}(\omega)}\approx 1/\tilde{F}(\omega)=[\tilde{F}(\omega)]^{-1}. \tag{2}$$

The approximation in Eq. (2) follows if the gain-feedback product is very large compared to unity, i.e., $|\tilde{F}(\omega)\tilde{G}(\omega)| \gg 1$. Thus, to a good approximation, it is possible to invert the transfer function of any passive linear circuit with this negative feedback circuit. This also implies through Eq. (1) that the well-known golden rule, $\tilde{A}(\omega)\approx\tilde{B}(\omega)$, holds under the same conditions. Equation (2) also implies that the negative feedback circuit shown in Fig. 1 can completely undo any deleterious effects, such as propagation delays, produced by a linear passive circuit [whose transfer function is identical to $\tilde{F}(\omega)$].

The group delay of the negative feedback circuit in the high gain-feedback limit is given by

$$\tau_{\tilde{T}(\omega)}=\frac{d\arg\tilde{T}(\omega)}{d\omega}\approx\frac{d\arg[1/\tilde{F}(\omega)]}{d\omega}$$

$$=-\frac{d\arg\tilde{F}(\omega)}{d\omega}=-\tau_{\tilde{F}(\omega)}. \tag{3}$$

This shows that the positive group delay from any linear passive circuit can, in principle, be completely canceled out by the negative group delay from a negative feedback circuit.

In Fig. 4 we show one example, where an RC low-pass filter is placed before the negative feedback circuit. The positive propagation delay $\tau_{\tilde{F}(\omega)}$ due to this RC low-pass circuit can, in principle, be completely canceled out by the negative group delay produced by the active circuit with the same RC element in its feedback loop.

It is important to note that this negative feedback scheme places a requirement on the gain-bandwidth product of the amplifier. For this active circuit to advance the waveform, it must have a large gain at all of the frequency components present in the signal. In particular, if we want to counteract a particular RC time delay, the amplifier must have a large gain at frequencies greater than $1/RC$. This limitation also effectively defines what is meant by an analytic waveform.

C. Kramers-Kronig relations necessitate superluminal group velocities, and Bode relations necessitate negative group delays

These counterintuitive results also follow quite generally from the Kramers-Kronig relations in the optical domain, and the analogous Bode relations in the electronic domain. It has been proven [10], starting from the principle of causality and the additional assumption of linearity, that superluminal group velocities in any medium must generally exist in some spectral region, and that for an amplifying medium, this spectral region must exist away from the regions with gain (i.e., in the transparent regions outside of the gain lines). Negative group delays in the electronic domain similarly follow generally from the Bode relations. Thus, causality itself necessitates the existence of these counterintuitive phenomena.

D. Energy transport by pulses in the optical and electronic domains

In the optical domain, there has been a debate concerning whether or not the velocity of energy transport by a wave packet can exceed c when the group velocity of the wave packet exceeds c. In the case of anomalous dispersion inside an absorption line, Sommerfeld and Brillouin showed that the energy velocity, defined as

$$v_{energy}\equiv\frac{\langle S\rangle}{\langle u\rangle}, \tag{4}$$

where $\langle S\rangle$ is the time-averaged Poynting vector and $\langle u\rangle$ is the time-averaged energy density of the electromagnetic wave, is different from the group velocity [11,12]. Although the group velocity in the region of absorptive anomalous dispersion exceeds c, they predicted that the energy velocity is less than c. However, experiments on picosecond laser pulse propagation in absorptive anomalous dispersive media have shown that these laser pulses travel with a superluminal group velocity, and not with the subluminal energy velocity of Sommerfeld and Brillouin [2]. Hence the physical meaning of this energy velocity is unclear.

When the optical medium possesses gain, as in the case of laserlike media with inverted atomic populations, the question arises as to whether or not to include the energy stored in the inverted atoms in the definition of $\langle u\rangle$ [13,14]. In regions of anomalous dispersion outside of the gain line and, in particular, in a spectral region where the group-velocity dispersion vanishes, a straightforward application of the Sommerfeld and Brillouin definition of the energy velocity would imply that the group and energy velocities both exceed *c*. The equality of these two kinds of wave velocities arises because the pulses of light are propagating inside a transparent medium with little dispersion. Additionally, in the case where the energy velocity is negative, the maximum in the pulse of energy leaves the exit face of the optical sample before the maximum in the pulse of energy enters the entrance face, just as for negative group velocities.

For an electronic circuit that produces negative group delays, the question of when the peak of the energy arrives can be answered by terminating the output port of Fig. 1 by a load resistor. The load resistor (not shown) will be heated up by the energy in the *output* pulse. It is obvious that the load resistor will then experience the maximum amount of heating when the peak of the Gaussian output pulse arrives at this resistor, and that this happens when the peak of the output voltage wave form arrives. For negative group delays, the load resistor will then heat up earlier than expected. The operational amplifier can supply the necessary energy to heat up the load resistor ahead of time. Hence, the negative group and the negative energy delays are identical in this case.

E. Preliminary data demonstrating the elimination of propagation delays from *RC* time constants

Using the circuit displayed in Fig. 4, we obtained the data shown in Fig. 5 of the output traces from a square wave input into an *RC* low-pass circuit, with (in the active response trace) and without (in the delayed response trace) the negative feedback circuit inserted after it. Clearly, the propagation delays on both the rising and falling edges of the delayed response trace (due to the *RC* time constant) have been almost completely eliminated by the negative feedback circuit. There is a ringing or overshoot phenomenon, associated with phase shifts in the feedback loop, accompanying the restoration of these sharp edges; however, this ringing can be reduced or eliminated by modifying the output poles of the amplifier. Furthermore, since the CMOS (complementary metal-oxide semiconductor) switching levels between logic states occur within 10% of 0 V for low-level signals, and within 90% of volt-level high-level signals, the observed ringing should not necessarily be deleterious for the purposes of computer speedup.

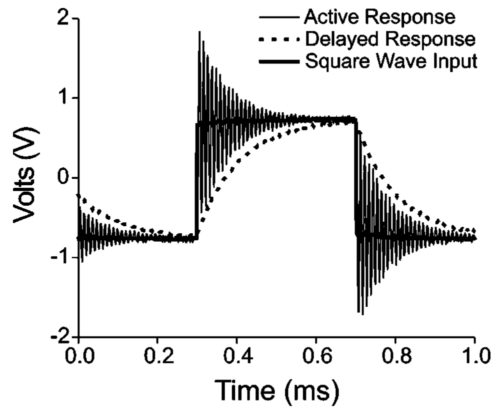

FIG. 5. V_{out} with (active response) and without (delayed response) the negative feedback, given the illustrated square wave input.

It seems from these data that both the *RC* time constants associated with transistor gates (the ‘‘latency’’ problem), and the *RC* propagation delays from wire interconnections can, in principle, be eliminated by negative feedback elements.

III. CONCLUSIONS

We have experimentally shown that is possible to advance a logic signal in time using a linear amplifier circuit with a negative feedback loop. We performed a theoretical analysis of the circuit and elaborated on the necessity of negative group delays as a consequence of the Bode relations, establishing an analogy between the optical and electronic domains. We also briefly addressed the problem of energy transport. Finally, we claimed that it may be possible to use a type of superluminal effect to compensate for deleterious time delays in microelectronics.

ACKNOWLEDGMENTS

We are indebted to Magnus Haakestad for giving us crucial suggestions at an early stage of this work. We also thank M. Mohajedi for his help. J.M.H. thanks the Instituto do Milenio de Informaçao Quântica, CAPES, CNPq, FAPEAL, PRONEX-NEON and CT PETRO for support.

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